

Calculation Policy

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Introduction

At Higham Primary School, we have a consistent approach to the teaching of written calculation methods in order to ensure continuity and progression across the school. The aim is for all children to have one written method for each of the four operations, which is reliable and efficient. This calculation policy has been written in line with the programmes of study taken from the National Curriculum for Mathematics (2014). It provides guidance on a structured and systematic approach to the teaching of calculation. The aim is for mental calculations and written procedures to be performed efficiently, fluently, accurately with understanding. Procedures and understanding are to be developed in tandem. End of the key stage expectations are explicit in the programme of study.

Mathematics Mastery

At the centre of the mastery approach to the teaching of mathematics is the belief that all children have the potential to succeed. They should have access to the same curriculum content and, rather than being extended with new learning, they should deepen their conceptual understanding by tackling challenging and varied problems. Similarly, with calculation strategies, children must not simply rote learn procedures but demonstrate their understanding of these procedures through the use of concrete materials and pictorial representations.

Mathematical Language

The 2014 National Curriculum is explicit in articulating the importance of children using the correct mathematical language as a central part of their learning (reasoning). Indeed, in certain year groups, the non-statutory guidance highlights the requirement for children to extend their language around certain concepts. It is therefore essential that teaching using the strategies outlined in this policy is accompanied by the use of appropriate and precise mathematical vocabulary. New vocabulary should be introduced in a suitable context (for example, with relevant real objects, apparatus, pictures or diagrams) and explained carefully. High expectations of the mathematical language used are essential, with teachers only accepting what is correct. The school agreed list of terminology is located at Appendix A to this document.

How to use the policy

This mathematics policy is a guide for all staff at Higham Primary School and has been adapted from work by the White Rose Mathematics Programme and NCETM. It is purposely set out as a progression of mathematical skills and not into year group phases to encourage a flexible approach to teaching and learning. It is expected that teachers will use their professional judgement as to when consolidation of existing skills is required or if to move onto the next concept. However, the focus must always remain on breadth and depth rather than accelerating through concepts. Children should not be extended with new learning before they are ready, they should deepen their conceptual understanding by tackling challenging and varied problems. All teachers have been given the scheme of work from the White Rose Maths Hub and are required to base their planning around their year group's modules and not to move onto a higher year group's scheme work. These modules use the Singapore Maths Methods and are affiliated to the workings of the 2014 Maths Programme of Study. Teachers can use any teaching resources that they wish to use and the policy does not recommend one set of resources over another, rather that, a variety of resources are used. For each of the four rules of number, different strategies are laid out, together with examples of what concrete materials can be used and how, along with suggested pictorial representations. The principle of the concrete-pictorial-abstract (CPA) approach [Make it, Draw it, Write it] is for children to have a true understanding of a mathematical concept, they need to master all three phases within a year group's scheme of work.





Additional Guidance

	EYFS / Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Addition	Combining two parts to make a whole: part whole model. Starting at the bigger number and counting onusing cubes. Regrouping to make 10 using ten frame.	Adding three single digits. Use of base 10 to combine two numbers.	Column method- regrouping. Using place value counters (up to 3 digits).	Column method- regrouping. (up to 4 digits)	Column method- regrouping. Use of place value counters for adding decimals.	Column method- regrouping. Abstract methods. Place value counters to be used for adding decimal numbers.
Subtraction	Taking away ones Counting back Find the difference Part whole model Make 10 using the ten frame	Counting back Find the difference Part whole model Make 10 Use of base 10	Column method with regrouping. (up to 3 digits using place value counters)	Column method with regrouping. (up to 4 digits)	Column method with regrouping. Abstract for whole numbers. Start with place value counters for decimals- with the same amount of decimal places.	Column method with regrouping. Abstract methods. Place value counters for decimals- with different amounts of decimal places.
Multiplication	Recognising and making equal groups. Doubling Counting in multiples Use cubes, Numicon and other objects in the classroom	Arrays- showing commutative multiplication Groups of	Arrays 2d × 1d using base 10 Column multiplication	Column multiplication Introduced with place value counters. (2 and 3 digit multiplied by 1 digit)	Column multiplication Abstract only but might need a repeat of year 4 first(up to 4 digit numbers multiplied by 1 or 2 digits)	Column multiplication Abstract methods (multidigit up to 4 digits by a 2 digit number)
Division	Sharing objects into groups Division as grouping e.g. I have 12 sweets and put them in groups of 3, how many groups? Use cubes and draw round 3 cubes at a time.	Division as grouping Division within arrays- linking to multiplication Repeated subtraction	Division with a remainder- using lollipop sticks, times tables facts and repeated subtraction. 2d divided by 1d using base 10 or place value counters Short division (up to 2digits by 1 digit)	Division with a remainder Short division (up to 3 digits by 1 digit- concrete and pictorial)	Short division (up to 4 digits by a 1 digit number including remainders)	Short division Long division with place value counters (up to 4 digits by a 2 digit number) Children should exchange into the tenths and hundredths column too



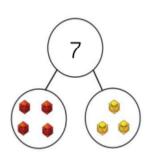


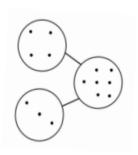
Models and Images

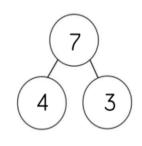
Below is an overview of the different models and images that can support the teaching of different concepts. These provide explanations of the benefits of using the models and show the links between different operations.



Part - Whole Model (Addition and Subtraction)





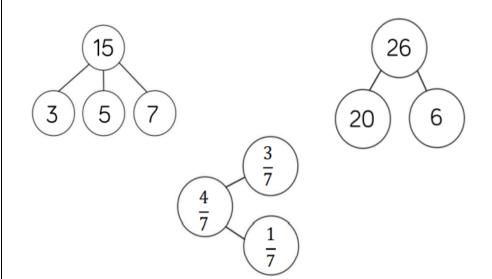


$$7 = 4 + 3$$

$$7 = 3 + 4$$

$$7 - 3 = 4$$

$$7 - 4 = 3$$



Benefits

This part-whole model supports children in their understanding of aggregation and partitioning. Due to its shape, it can be referred to as a cherry part-whole model.

When the parts are complete and the whole is empty, children use aggregation to add the parts together to find the total.

When the whole is complete and at least one of the parts is empty, children use partitioning (a form of subtraction) to find the missing part.

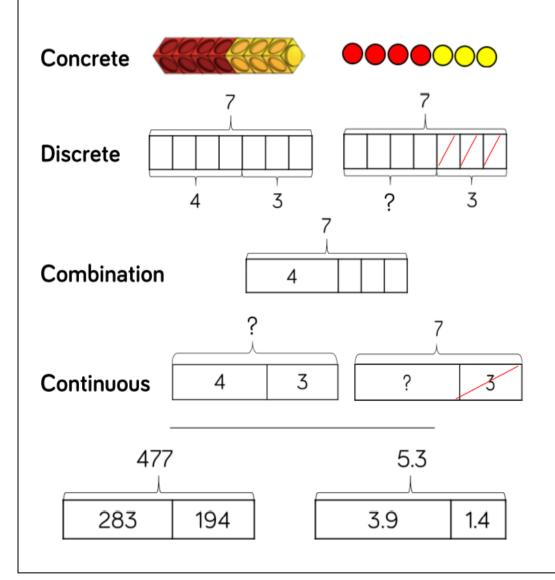
Part-whole models can be used to partition a number into two or more parts, or to help children to partition a number into tens and ones or other place value columns.

In KS2, children can apply their understanding of the part-whole model to add and subtract fractions, decimals and percentages.





Bar Model - Single (Addition and Subtraction)



Benefits

The single bar model is another type of a part-whole model that can support children in representing calculations to help them unpick the structure.

Cubes and counters can be used in a line as a concrete representation of the bar model.

Discrete bar models are a good starting point with smaller numbers. Each box represents one whole.

The combination bar model can support children to calculate by counting on from the larger number. It is a good stepping stone towards the continuous bar model.

Continuous bar models are useful for a range of values. Each rectangle represents a number. The question mark indicates the value to be found.

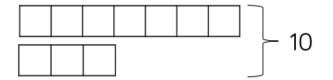
In KS2, children can use bar models to represent larger numbers, decimals and fractions.



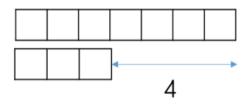


Bar Model – Multiple (Addition and Subtraction)

Discrete

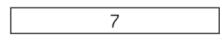


$$7 + 3 = 10$$



$$7 - 3 = 4$$

Continuous





$$7 - 3 = 4$$

2,394



$$2,394 - 1,014 = 1,380$$

Benefits

The multiple bar model is a good way to compare quantities whilst still unpicking the structure.

Two or more bars can be drawn, with a bracket labelling the whole positioned on the right hand side of the bars. Smaller numbers can be represented with a discrete bar model whilst continuous bar models are more effective for larger numbers.

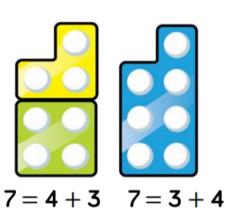
Multiple bar models can also be used to represent the difference in subtraction. An arrow can be used to model the difference.

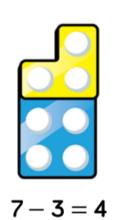
When working with smaller numbers, children can use cubes and a discrete model to find the difference. This supports children to see how counting on can help when finding the difference.

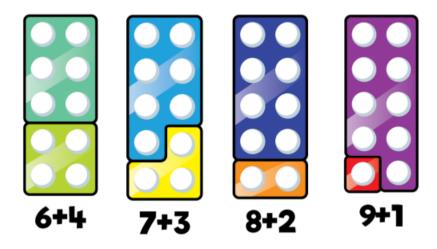




Number Shapes - Numicon (Addition and Subtraction)







Benefits

Number shapes can be useful to support children to subitise numbers as well as explore aggregation, partitioning and number bonds.

When adding numbers, children can see how the parts come together making a whole. As children use number shapes more often, they can start to subitise the total due to their familiarity with the shape of each number.

When subtracting numbers, children can start with the whole and then place one of the parts on top of the whole to see what part is missing. Again, children will start to be able to subitise the part that is missing due to their familiarity with the shapes.

Children can also work systematically to find number bonds. As they increase one number by 1, they can see that the other number decreases by 1 to find all the possible number bonds for a number.





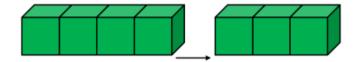
Cubes (Addition and Subtraction)



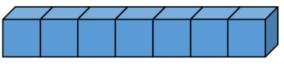
$$7 = 4 + 3$$

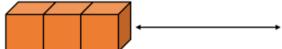


$$7 = 3 + 4$$



$$7 - 3 = 4$$





$$7 - 3 = 4$$

Benefits

Cubes can be useful to support children with the addition and subtraction of one-digit numbers.

When adding numbers, children can see how the parts come together to make a whole. Children could use two different colours of cubes to represent the numbers before putting them together to create the whole.

When subtracting numbers, children can start with the whole and then remove the number of cubes that they are subtracting in order to find the answer. This model of subtraction is reduction, or take away.

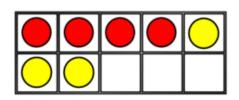
Cubes can also be useful to look at subtraction as difference. Here, both numbers are made and then lined up to find the difference between the numbers.

Cubes are useful when working with smaller numbers but are less efficient with larger numbers as they are difficult to subitise and children may miscount them.

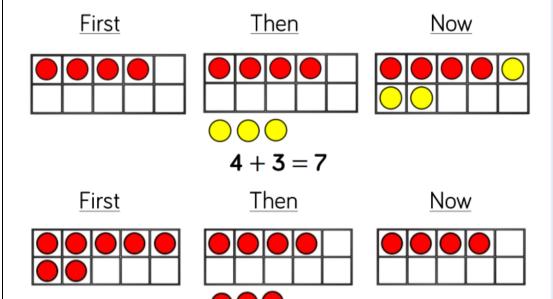




Tens Frame - Within 10 (Addition and Subtraction)



$$4+3=7$$
 4 is a part.
 $3+4=7$ 3 is a part.
 $7-3=4$ 7 is the whole.
 $7-4=3$



7 - 3 = 4

Benefits

When adding and subtracting within 10, the ten frame can support children to understand the different structures of addition and subtraction.

Using the language of parts and wholes represented by objects on the ten frame introduces children to aggregation and partitioning.

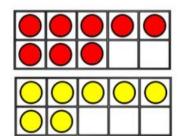
Aggregation is a form of addition where parts are combined together to make a whole. Partitioning is a form of subtraction where the whole is split into parts. Using these structures, the ten frame can enable children to find all the number bonds for a number.

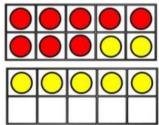
Children can also use ten frames to look at augmentation (increasing a number) and take-away (decreasing a number). This can be introduced through a first, then, now structure which shows the change in the number in the 'then' stage. This can be put into a story structure to help children understand the change e.g. First, there were 7 cars. Then, 3 cars left. Now, there are 4 cars.

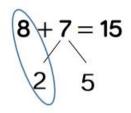


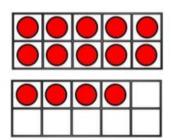


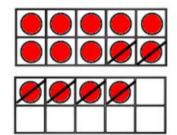
Tens Frame - Within 20 (Addition and Subtraction)

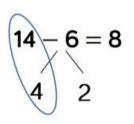


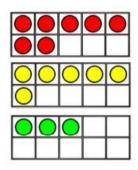


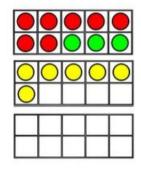


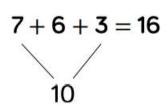












Benefits

When adding two single digits, children can make each number on separate ten frames before moving part of one number to make 10 on one of the ten frames. This supports children to see how they have partitioned one of the numbers to make 10, and makes links to effective mental methods of addition.

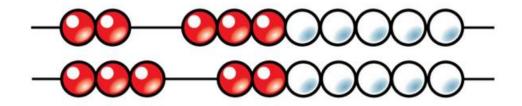
When subtracting a one-digit number from a two-digit number, firstly make the larger number on 2 ten frames. Remove the smaller number, thinking carefully about how you have partitioned the number to make 10, this supports mental methods of subtraction.

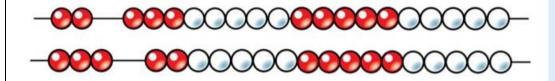
When adding three single-digit numbers, children can make each number on 3 separate 10 frames before considering which order to add the numbers in. They may be able to find a number bond to 10 which makes the calculation easier. Once again, the ten frames support the link to effective mental methods of addition as well as the importance of commutativity.

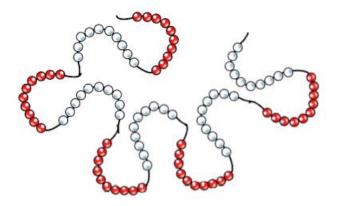




Bead Strings (Addition and Subtraction)







Benefits

Different sizes of bead strings can support children at different stages of addition and subtraction.

Bead strings to 10 are very effective at helping children to investigate number bonds up to 10.

They can help children to systematically find all the number bonds to 10 by moving one bead at a time to see the different numbers they have partitioned the 10 beads into e.g. 2 + 8 = 10, move one bead, 3 + 7 = 10.

Bead strings to 20 work in a similar way but they also group the beads in fives. Children can apply their knowledge of number bonds to 10 and see the links to number bonds to 20.

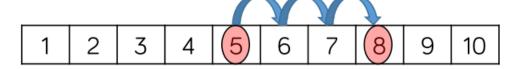
Bead strings to 100 are grouped in tens and can support children in number bonds to 100 as well as helping when adding by making ten. Bead strings can show a link to adding to the next 10 on number lines which supports a mental method of addition.



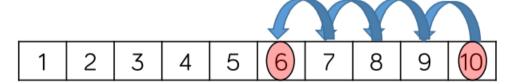


Number Tracks (Addition and Subtraction)

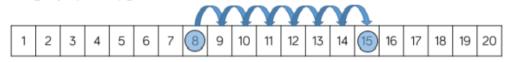




$$10 - 4 = 6$$



$$8 + 7 = 15$$



Benefits

Number tracks are useful to support children in their understanding of augmentation and reduction.

When adding, children count on to find the total of the numbers. On a number track, children can place a counter on the starting number and then count on to find the total.

When subtracting, children count back to find their answer. They start at the minuend and then take away the subtrahend to find the difference between the numbers

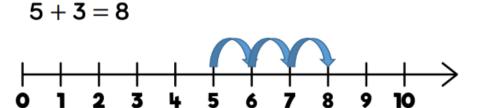
Number tracks can work well alongside ten frames and bead strings which can also model counting on or counting back.

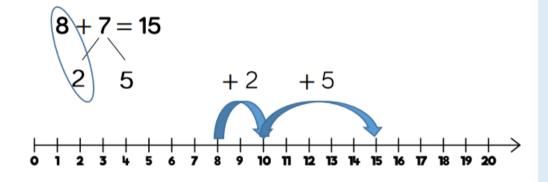
Playing board games can help children to become familiar with the idea of counting on using a number track before they move on to number lines.

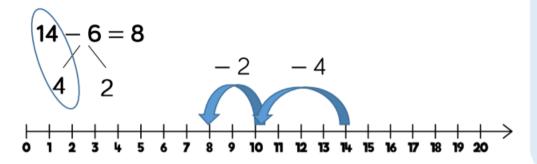




Number Lines Labelled (Addition and Subtraction)







Benefits

Labelled number lines support children in their understanding of addition and subtraction as augmentation and reduction.

Children can start by counting on or back in ones, up or down the number line. This skill links directly to the use of the number track.

Progressing further, children can add numbers by jumping to the nearest 10 and then jumping to the total. This links to the making 10 method which can also be supported by ten frames. The smaller number is partitioned to support children to make a number bond to 10 and to then add on the remaining part.

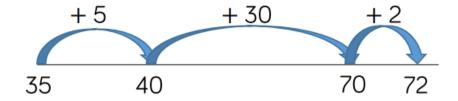
Children can subtract numbers by firstly jumping to the nearest 10. Again, this can be supported by ten frames so children can see how they partition the smaller number into the two separate jumps.



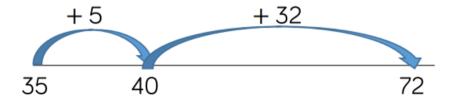


Number Lines Blank (Addition and Subtraction)

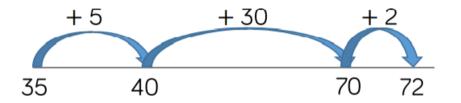




$$35 + 37 = 72$$



$$72 - 35 = 37$$



Benefits

Blank number lines provide children with a structure to add and subtract numbers in smaller parts.

Developing from labelled number lines, children can add by jumping to the nearest 10 and then adding the rest of the number either as a whole or by adding the tens and ones separately.

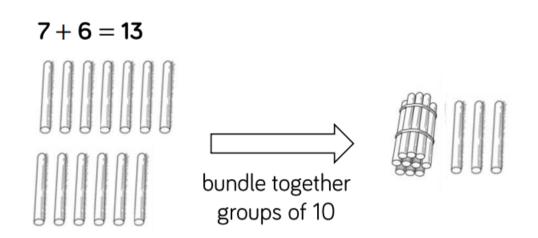
Children may also count back on a number line to subtract, again by jumping to the nearest 10 and then subtracting the rest of the number.

Blank number lines can also be used effectively to help children subtract by finding the difference between numbers. This can be done by starting with the smaller number and then counting on to the larger number. They then add up the parts they have counted on to find the difference between the numbers.

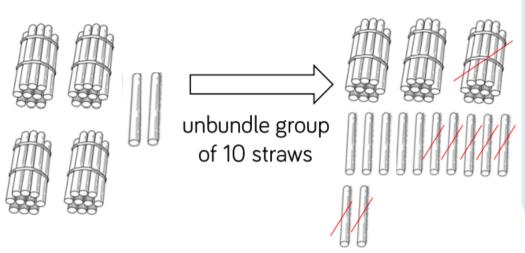




Straws (Addition and Subtraction)



$$42 - 17 = 25$$



Benefits

Straws are an effective way to support children in their understanding of exchange when adding and subtracting 2-digit numbers.

Children can be introduced to the idea of bundling groups of ten when adding smaller numbers and when representing 2-digit numbers. Use elastic bands or other ties to make bundles of ten straws.

When adding numbers, children bundle a group of 10 straws to represent the exchange from 10 ones to 1 ten. They then add the individual straws (ones) and bundles of straws (tens) to find the total.

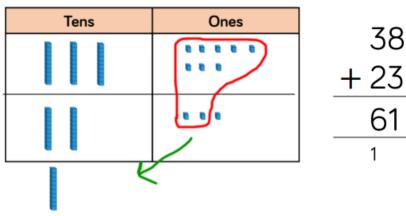
When subtracting numbers, children unbundle a group of 10 straws to represent the exchange from 1 ten to 10 ones.

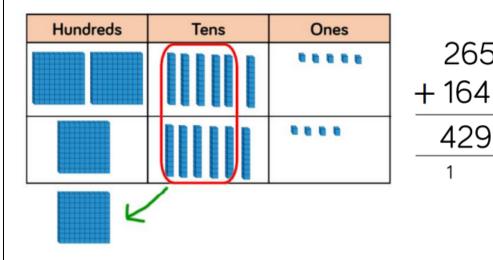
Straws provide a good stepping stone to adding and subtracting with Base 10/Dienes.





Base 10/Dienes (Addition)





38 Using Base 10 or Dienes is an effective way to support children's understanding of column addition. It is important that children write out their calculations

265

429

Children should first add without an exchange before moving on to addition with exchange.. The representation becomes less efficient with larger numbers due to the size of Base 10. In this case, place value counters may be the better model to use.

alongside using or drawing Base 10 so they can see the

clear links between the written method and the model.

When adding, always start with the smallest place value column. Here are some questions to support children. How many ones are there altogether? Can we make an exchange? (Yes or No)

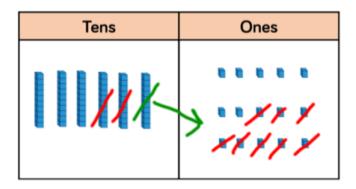
How many do we exchange? (10 ones for 1 ten, show exchanged 10 in tens column by writing 1 in column) How many ones do we have left? (Write in ones column) Repeat for each column.

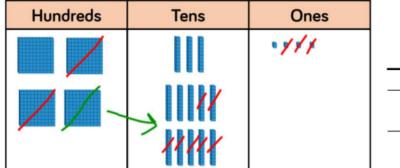
Benefits





Base 10/Dienes (Subtraction)





 $-\frac{3435}{262}$

Benefits

Using Base 10 or Dienes is an effective way to support children's understanding of column subtraction. It is important that children write out their calculations alongside using or drawing Base 10 so they can see the clear links between the written method and the model.

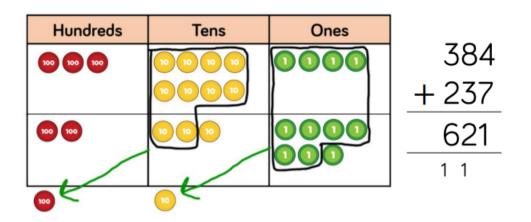
Children should first subtract without an exchange before moving on to subtraction with exchange. When building the model, children should just make the minuend using Base 10, they then subtract the subtrahend. Highlight this difference to addition to avoid errors by making both numbers. Children start with the smallest place value column. When there are not enough ones/tens/hundreds to subtract in a column, children need to move to the column to the left and exchange e.g. exchange 1 ten for 10 ones. They can then subtract efficiently.

This model is efficient with up to 4-digit numbers. Place value counters are more efficient with larger numbers and decimals.





Place Value Counters (Addition)



Ones	Tenths	Hundredths	
000	01 01 01	0.01 0.01	3.65
	0.1 0.1 0.1	0.01 0.01	+ 2.41
00	01 01 01	0.01	6.06
	01		1

Benefits

Using place value counters is an effective way to support children's understanding of column addition. It is important that children write out their calculations alongside using or drawing counters so they can see the clear links between the written method and the model.

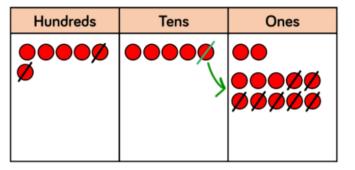
Children should first add without an exchange before moving on to addition with exchange. Different place value counters can be used to represent larger numbers or decimals. If you don't have place value counters, use normal counters on a place value grid to enable children to experience the exchange between columns.

When adding money, children can also use coins to support their understanding. It is important that children consider how the coins link to the written calculation especially when adding decimal amounts.





Place Value Counters (Subtraction)



$$\begin{array}{r}
65^{1} \\
-207 \\
445
\end{array}$$

Thousands	Hundreds	Tens	Ones	- 1
	100 100 100			³ 4 357
6	@@@Ø			– 2735
'	QQ			1622

Benefits

Using place value counters is an effective way to support children's understanding of column subtraction. It is important that children write out their calculations alongside using or drawing counters so they can see the clear links between the written method and the model.

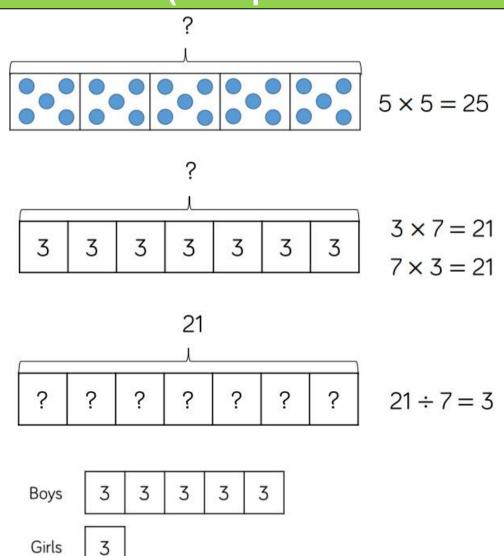
Children should first subtract without an exchange before moving on to subtraction with exchange. If you don't have place value counters, use normal counters on a place value grid to enable children to experience the exchange between columns.

When building the model, children should just make the minuend using counters, they then subtract the subtrahend. Children start with the smallest place value column. When there are not enough ones/tens/hundreds to subtract in a column, children need to move to the column to the left and exchange e.g. exchange 1 ten for 10 ones. They can then subtract efficiently.





Bar Model (Multiplication and Division)



Benefits

Children can use the single bar model to represent multiplication as repeated addition. They could use counters, cubes or dots within the bar model to support calculation before moving on to placing digits into the bar model to represent the multiplication.

Division can be represented by showing the total of the bar model and then dividing the bar model into equal groups.

It is important when solving word problems that the bar model represents the problem.

Sometimes, children may look at scaling problems. In this case, more than one bar model is useful to represent this type of problem, e.g. There are 3 girls in a group. There are 5 times more boys than girls. How many boys are there?

The multiple bar model provides an opportunity to compare the groups.



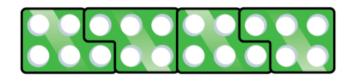


Number Shapes (Multiplication and Division)



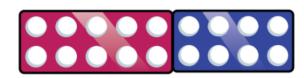
$$5 \times 4 = 20$$

 $4 \times 5 = 20$

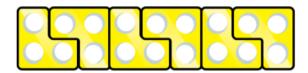


$$5 \times 4 = 20$$

 $4 \times 5 = 20$



$$18 \div 3 = 6$$



Benefits

Number shapes support children's understanding of multiplication as repeated addition.

Children can build multiplications in a row using the number shapes. When using odd numbers, encourage children to interlock the shapes so there are no gaps in the row. They can then use the tens number shapes along with other necessary shapes over the top of the row to check the total. Using the number shapes in multiplication can support children in discovering patterns of multiplication e.g. odd \times odd = even, odd \times even = odd, even \times even = even.

When dividing, number shapes support children's understanding of division as grouping. Children make the number they are dividing and then place the number shape they are dividing by over the top of the number to find how many groups of the number there are altogether e.g. There are 6 groups of 3 in 18.





Bead Strings (Multiplication and Division)



$$5 \times 3 = 15$$

 $3 \times 5 = 15$
 $15 \div 3 = 5$



$$5 \times 3 = 15$$

 $3 \times 5 = 15$
 $15 \div 5 = 3$



$$4 \times 5 = 20$$

 $5 \times 4 = 20$ $20 \div 4 = 5$

Benefits

Bead strings to 100 can support children in their understanding of multiplication as repeated addition. Children can build the multiplication using the beads. The colour of beads supports children in seeing how many groups of 10 they have, to calculate the total more efficiently.

Encourage children to count in multiples as they build the number e.g. 4, 8, 12, 16, 20.

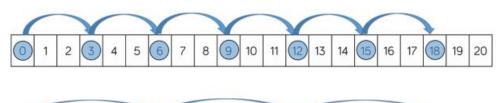
Children can also use the bead string to count forwards and backwards in multiples, moving the beads as they count.

When dividing, children build the number they are dividing and then group the beads into the number they are dividing by e.g. 20 divided by 4 – Make 20 and then group the beads into groups of four. Count how many groups you have made to find the answer.





Number Tracks (Multiplication and Division)





$$6 \times 3 = 18$$

 $3 \times 6 = 18$



$$18 \div 3 = 6$$

Benefits

Number tracks are useful to support children to count in multiples, forwards and backwards. Moving counters or cubes along the number track can support children to keep track of their counting. Translucent counters help children to see the number they have landed on whilst counting.

When multiplying, children place their counter on 0 to start and then count on to find the product of the numbers.

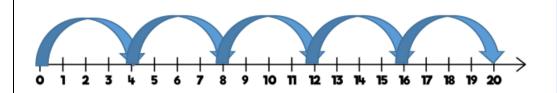
When dividing, children place their counter on the number they are dividing and the count back in jumps of the number they are dividing by until they reach 0. Children record how many jumps they have made to find the answer to the division.

Number tracks can be useful with smaller multiples but when reaching larger numbers they can become less efficient.





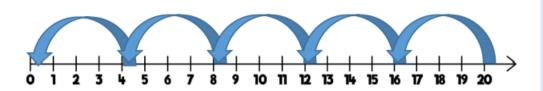
Number Lines Labelled (Multiplication and Division)





$$4 \times 5 = 20$$

 $5 \times 4 = 20$



$$20 \div 4 = 5$$

Benefits

Labelled number lines are useful to support children to count in multiples, forwards and backwards as well as calculating single-digit multiplications.

When multiplying, children start at 0 and then count on to find the product of the numbers.

When dividing, start at the number they are dividing and the count back in jumps of the number they are dividing by until they reach 0.

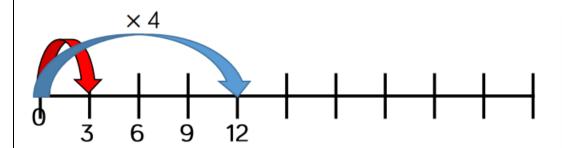
Children record how many jumps they have made to find the answer to the division.

Labelled number lines can be useful with smaller multiples, however they become inefficient as numbers become larger due to the required size of the number line.





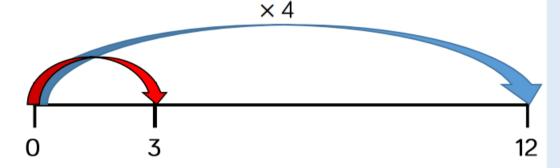
Number Lines Blank (Multiplication and Division)



A red car travels 3 miles.

A blue car 4 times further.

How far does the blue car travel?



A blue car travels 12 miles.
A red car 4 times less.
How far does the red car travel?

Benefits

Children can use blank number lines to represent scaling as multiplication or division.

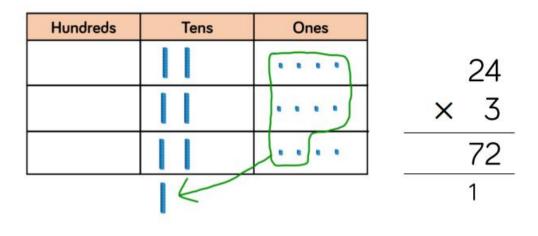
Blank number lines with intervals can support children to represent scaling accurately. Children can label intervals with multiples to calculate scaling problems.

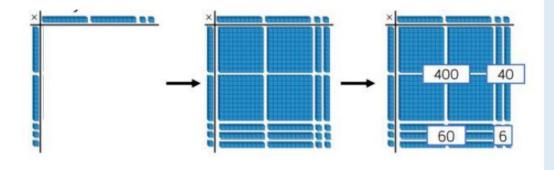
Blank number lines without intervals can also be used for children to represent scaling.





Base 10/Dienes (Multiplication)





Benefits

Using Base 10 or Dienes is an effective way to support children's understanding of column multiplication. It is important that children write out their calculation alongside the equipment so they can see how the concrete and written representations match.

As numbers become larger in multiplication or the amounts of groups becomes higher, Base 10 / Dienes becomes less efficient due to the amount of equipment and number of exchanges needed.

Base 10 also supports the area model of multiplication well. Children use the equipment to build the number in a rectangular shape which they then find the area of by calculating the total value of the pieces This area model can be linked to the grid method or the formal column method of multiplying 2-digits by 2-digits.



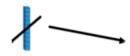


Base 10/Dienes (Division)



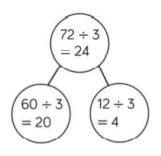


$$68 \div 2 = 34$$



Tens	Ones	
	•	

$$72 \div 3 = 24$$



Benefits

Using Base 10 or Dienes is an effective way to support children's understanding of division.

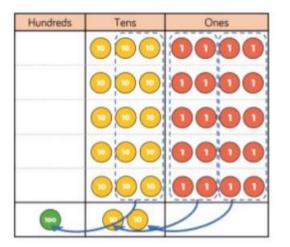
When numbers become larger, it can be an effective way to move children from representing numbers as ones towards representing them as tens and ones in order to divide. Children can then share the Base 10/ Dienes between different groups e.g. by drawing circles or by rows on a place value grid.

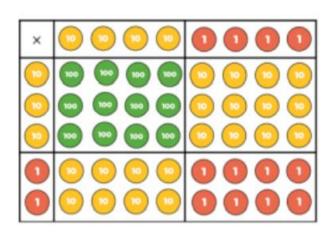
When they are sharing, children start with the larger place value and work from left to right. If there are any left in a column, they exchange e.g. one ten for ten ones. When recording, encourage children to use the part—whole model so they can consider how the number has been partitioned in order to divide. This will support them with mental methods.





Place Value Counters (Multiplication)





Benefits

Using place value counters is an effective way to support children's understanding of column multiplication. It is important that children write out their calculation alongside the equipment so they can see how the concrete and written match.

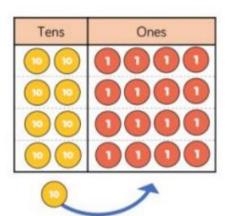
As numbers become larger in multiplication or the amounts of groups becomes higher, Base 10 / Dienes becomes less efficient due to the amount of equipment and number of exchanges needed The counters should be used to support the understanding of the written method rather than support the arithmetic.

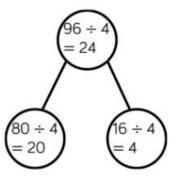
Place value counters also support the area model of multiplication well. Children can see how to multiply 2-digit numbers by 2-digit numbers.

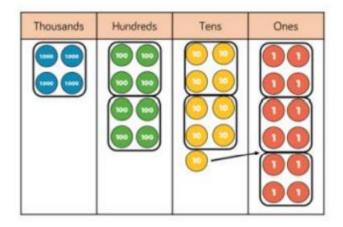




Place Value Counters (Division)







1223 4 489¹2

Benefits

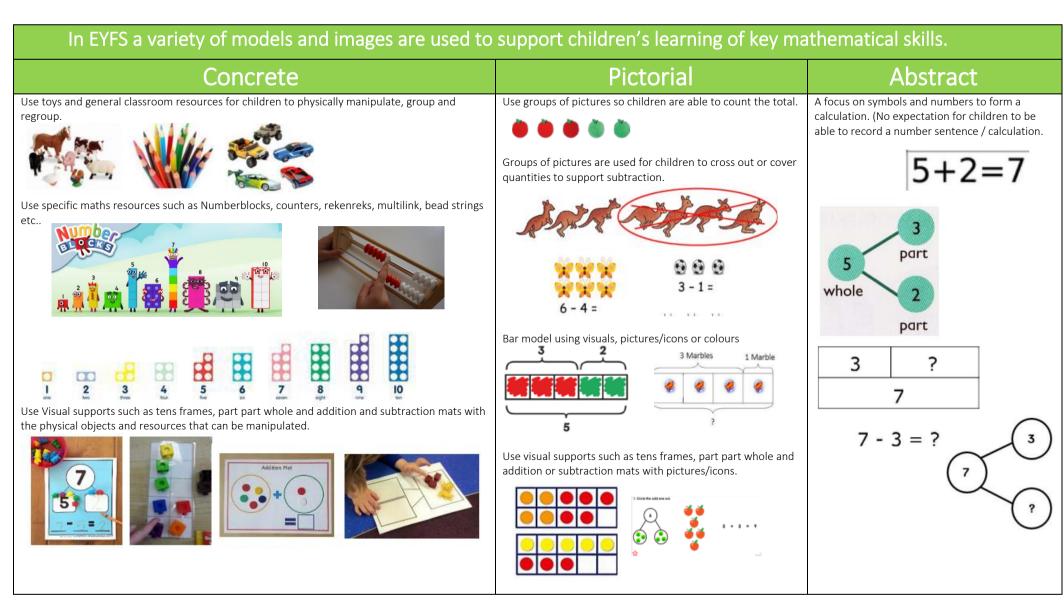
Using place value counters is an effective way to support children's understanding of division.

When working with smaller numbers, children can use place value counters to share between groups. They start by sharing the larger place value column and work from left to right. If there are any counters left over once they have been shared, they exchange the counter e.g. exchange one ten for ten ones. This method can be linked to the part-whole model to support children to show their thinking.

Place value counters also support children's understanding of short division by grouping the counters rather than sharing them. Children work from left to right through the place value columns and group the counters in the number they are dividing by. If there are any counters left over after they have been grouped, they exchange the counter e.g. exchange one hundred for ten tens.











Addition Key Language: sum, total, parts and wholes, plus, add, altogether, more, 'is equal to' ' is the same as' **Pictorial Abstract** Concrete Combining two parts to make a whole (use other Children to represent the cubes using dots or crosses. 4 + 3 = 7resources too e.g. eggs, shells, teddy bears, cars). They could put each part on a part whole model too. Four is a part, 3 is a part and the whole is seven Counting on using number lines using cubes, bead A bar model which encourages the children to count on, The abstract number line: strings or Numicon. Start at the biggest number. rather than count all. What is 2 more than 4? What is the sum of 2 and 4? What is the total of 4 and 2? 4 + 2 Place the larger number in your head and count on the smaller number to help you find the answer.





Addition

Key Language: sum, total, parts and wholes, plus, add, altogether, more, 'is equal to' ' is the same as'				
Concrete	Pictorial	Abstract		
Regrouping to make 10; using ten frames and counters/cubes or using Numicon. 6 + 5	Children to draw the ten frame and counters/cubes.	Children to develop an understanding of equality e.g. $6 + \square = 11$ $6 + 5 = 5 + \square$		
	Regroup or partition the smaller number using the part-part whole model to make 10	$6+3-3+\square$ $6+5=\square+4$ $7+4=11$		
	9 + 5 = 14	If I am at 7, how many more do I need to make 10? How many more do I need to add on now?		
Adding three single digits 4 + 7 + 6= 17 Put 4 and 6 together to make 10. Add on 7.		Combine the two numbers that make 10 and then add on the remainder. $4 + 7 + 6 = 10 + 7$		
Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.	Add together three groups of objects. Draw a picture to recombine the groups to make 10.	= [17]		
TO + O using base 10. Continue to develop understanding of partitioning and place value. 41 + 8	Children to represent the base 10 e.g. lines for tens and dot/crosses for ones.	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$		





Addition

Key Language: sum, total, parts and wholes, plus, add, altogether, more, 'is equal to' ' is the same as'				
Concrete	Pictorial	Abstract		
TO + TO using base 10. Continue to develop understanding of partitioning and place value. 36 + 25	Children to represent the base 10 in a place value chart.	Looking for ways to make 10. 36 + 25 = 30 + 20 = 50 5 + 5 = 10 1 5 $50 + 10 + 1 = 61$ 36 Formal method: $\frac{+25}{61}$		
Column method with regrouping. Use of place value counters to add HTO + TO, HTO + HTO etc. When there are 10 ones in the 1s column- we exchange for 1 ten, when there are 10 tens in the 10s column- we exchange for 1 hundred. As children move onto decimals, money and decimal place value counters can be used to support learning	Children to represent the counters in a place value chart, circling when they make an exchange. 100s 10s 1s NB Addition of money needs to have £ or p added separately.	100+40+6 500+20+7 600 + 70 + 3 = 673 As the children progress, they will move from the expanded to the compacted method. 146 + 527 673 1 As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here.		





Addition Key Language: sum, total, parts and wholes, plus, add, altogether, more, 'is equal to' ' is the same as'				
Concrete		Pictorial	Abstract	
Conceptual variation; different ways to ask children to solve 21 + 34				
? 21 34 ? 21 34	Word problems: In year 3, there are 21 child and in year 4, there are 34 children. How many children in total? 21 + 34 = 55. Prove it	+34	21 + 34 = = 21 + 34 he sum of twenty-one four.	Missing digit problems: 10 1 2 2 7 5

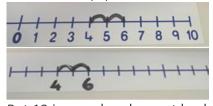




<u>Subtraction</u>				
Key Language: take away, less than, the difference, subtract, minus, fewer, decrease				
Concrete	Pictorial	Abstract		
Physically taking away and removing objects from a whole (ten frames, Numicon, cubes and other items such as beanbags could be used). $4-3=1$ 0 0 0 0	Children to draw the concrete resources they are using and cross out the correct amount. The bar model can also be used.	4-3= 		
Counting back (using number lines or number tracks) children start with 6 and count back 2. $6-2=4$ Make the larger number, then move the beads along	Children to represent what they see pictorially e.g.	Children to represent the calculation on a number line or number track and show their jumps. Encourage children to use an empty number line		

the bead string as you count backwards in ones.





Put 13 in your head, count back 4. What number are you at? Use your fingers to help you.





Subtraction

Key Language: take away, less than, the difference, subtract, minus, fewer, decrease					
Concrete	Pictorial	Abstract			
Finding the difference (using cubes, Numicon or Cuisenaire rods, other objects can also be used).	Children to draw the cubes/other concrete	Find the difference between 8 and 5.			
Calculate the difference between 8 and 5. Use basic bar models with	objects which they have used or use the bar model to illustrate what they need to calculate. Lisa is 13 years old. Her sister is 22 years old. Find the difference in age between them.	8-5, the difference is Children to explore why $9-6=8-5=7-4$ have the same difference.			
objects to find the difference.	Models 13 ? Lisa Sister 22	Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches.			
Part-part whole model - link to addition- use the part whole model to help explain the inverse between addition and subtraction. If 10 is the whole and 6 is one of the parts. What is the other part? 10 - 6 =	Use a pictorial representation of objects to show the part-part whole model.	Move to using numbers within the part whole model.			





Subtraction

Key Language: take away, less than, the difference, subtract, minus, fewer, decrease					
Concrete	Pictorial	Abstract			
Making 10 using ten frames. 14 – 5 - 4 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1	Children to present the ten frame pictorially and discuss what they did to make 10.	Children to show how they can make 10 by partitioning the subtrahend. $14 - 5 = 9$ $4 \qquad 1$ $14 - 4 = 10$ $10 - 1 = 9$			
Column method without regrouping using base 10. 48-7	Children to represent the base 10 pictorially.	Column method or children could count back 7. 4 8 - 7 4 1			
Column method using base 10 and having to exchange. 41 – 26 10s	Represent the base 10 pictorially, remembering to show the exchange.	Formal column method. Children must understand that when they have exchanged the 10 they still have 41 because 41 = 30 + 11.			





<u>Subtraction</u>						
Key Language: take away, less than, the difference, subtract, minus, fewer, decrease						
Concrete		Pict	torial	Abstract		
Column method using place value counters. 100s 10s 1s 100s 10s 1s 100s 10s 1s 100s 10s 1s 110s 10s 1s	00s 10s 1s	Represent the place value counters pictorially; remembering to show what has been exchanged.	100s 10s 1s	Formal column method. Children must understand what has happened when they have crossed out digits.		
Conceptual variation; differ	rent ways	to ask children to	o solve 391 - 186			
	• •	Timmy spent £186. How	= 391 – 186	Missing digit calculations		
	much more did Calculate the dif and 186.	ference between 391	391 -186 —— What is 186 less than 391?	3 9 <u>6</u> 0 5		
186 ?						





Multiplication

Key Language: double, times, multiplied by, the product of, groups of, lots of, equal groups					
Concrete	Pictorial	Abstract			
Repeated grouping/repeated addition 3 × 4 4 + 4 + 4 There are 3 equal groups, with 4 in each group. Use different objects to add equal groups	Children to represent the practical resources in a picture and use a bar model.	3 × 4 = 12 4 + 4 + 4 = 12			
Number lines to show repeated groups 3 × 4 Cuisenaire rods can be used too.	Represent this pictorially alongside a number line e.g.:	Abstract number line showing three jumps of four. 3 × 4 = 12			





Multiplication

Key Language: double, times, multiplied by, the product of, groups of, lots of, equal groups						
Concrete	Pictorial	Abstract				
Use arrays to illustrate commutativity counters and other objects can also be used. 2 × 5 = 5 × 2 Create arrays to show multiplication sentences.	Draw arrays in different rotations to find commutative multiplication sentences. Children to represent the arrays pictorially. Link arrays to area of rectangles	Children to be able to use an array to write a range of calculations e.g. $10 = 2 \times 5$ $5 \times 2 = 10$ $2 + 2 + 2 + 2 + 2 = 10$ $10 = 5 + 5$				
Partition to multiply using Numicon, base 10 or Cuisenaire rods. 4 × 15	Children to represent the concrete manipulatives pictorially.	Children to be encouraged to show the steps they have taken. 10 5 10 \times 4 = 40 5 \times 4 = 20 40 + 20 = 60				





Multiplication

Key Language: double, times, multiplied by, the product of, groups of, lots of, equal groups

Key Language: double, times, multiplied by, the product of, groups of, lots of, equal groups						
Concrete	Pictorial	Abstract				
Formal Column methods	Children can represent the work they have done with	Start with				
Grid method - link with arrays to first introduce.	place value counters/base 10 in a way that they	multiplying by 🗶 30 5				
x 10 3 4 rows of 10	understand.	one digit numbers 7 210 35				
4 rows of 3		and showing the 210 + 35 = 245				
	They can draw the counters, using colours to show	clear addition				
Use Base 10 to move towards a more compact method.	different amounts or just use circles in the different	alongside the grid.				
X T U	columns to show their thinking as shown below.					
4 rows of 13	24 × 3 = 72	Moving forward, multiply by a 2 digit				
	X 20 4	number showing the different rows within				
Use place value counters to show how we are finding	2 00 0000	the grid method.				
groups of a number.	5 20 2000	10 8 X 1000 300 40 2				
We are multiplying by	00 0000					
Calculations 4 x 126 4 so we need 4 rows.	60 12	20 10000 3000 400 25				
	60	3 30 24 8 8000 2400 320 16				
© Galculations Fill each row with	72					
4 x 126 126.	100s 10s 1s	Move to formal written method, children to				
	,	line up numbers clearly in columns.				
	00 000	6 x 23 =				
Add up each column, starting with the ones making any	00 000	23				
exchanges needed.	000 0000					
0 0000	0 00000	X 0 130				
	2 2 8	<u>× 6</u> 138 1 1				
4 × 126 = 504	3. 4.					
4 ^ 120 - 304						



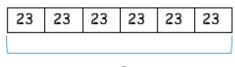


Multiplication

Key Language: double, times, multiplied by, the product of, groups of, lots of, equal groups

Concrete	Pictorial			Д	bs	tract				
Long multiplication using expanded method, then leading in When children start to multiply $3d \times 3d$ and $4d \times 2d$ etc., the lift it helps children can write out what they are solving next	ey should be confident with the abstract:	18 x <u>13</u> 24 (3 x 8) 30 (3 x 10)) 80 (10 x 8) <u>100</u> (10 x 10) 234	+ 4	2 2 2	7 6 1 1 4 0	2 0 0	× -7 -1 2 -4 3 2 1 1	4 8 2	6 4 0	

Conceptual variation; different ways to ask children to solve 6 × 23



3

Mai had to swim 23 lengths, 6 times
a week.
How many lengths did she swim in
one week?

With the counters, prove that 6 x 23 = 138

Find the product of 6 and 23

$$6 \times 23 =$$
 $= 6 \times 23$

6
23

× 23 × 6

What is the calculation? What is the product?

100s	10s	1s
	000000	000





<u>Division</u>						
Key Language: share group, divide, divid	Key Language: share group, divide, divided by, half					
Concrete	Pictorial	Abstract				
Division as sharing using a range of objects. 6 ÷ 2	Represent the sharing pictorially.	6 ÷ 2 = 3				
		3 3				
	?	Children should also be encouraged to use their 2 times tables facts.				
Repeated subtraction using Cuisenaire rods above a ruler. 6 ÷ 2	Children to represent repeated subtraction pictorially.	Abstract number line to represent the equal groups that have been subtracted.				
	00000006	-Z -2 -2 -2 3 4 5 6 3 groups				
3 groups of 2						





<u>DIVISION</u>						
Key Language: share group, divide, divided by, half						
Concrete	Pictorial	Abstract				
Division as grouping - Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.	Use a number line to show jumps in groups. The number of jumps equals the number of groups. Use bar modelling to aid solving division problems. Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.	10 ÷ 5 = 2 Divide 10 into 5 groups. How many are in each group?				
Division with arrays Link division to multiplication by creating an array and thinking about the number sentences that can be created. $15 \div 3 = 5 \qquad 5 \times 3 = 15$ $15 \div 5 = 3 \qquad 3 \times 5 = 15$	Draw an array and use lines to split the array into groups to make multiplication and division sentences.	Find the inverse of multiplication and division sentences by creating four linking number sentences. $5 \times 3 = 15$ $3 \times 5 = 15$ $15 \div 5 = 3$ $15 \div 3 = 5$				





Key Language: share group, divide, divided by, half						
Concrete	Pictorial	Abstract				
Division with remainders Divide objects between groups and see how much is left over 2d ÷ 1d with remainders using lollipop sticks. Cuisenaire rods, above a ruler can also be used. 13 ÷ 4 = Use of lollipop sticks to form wholes- squares are made because we are dividing by 4. There are 3 whole squares, with 1 left over.	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder. Draw dots and group them to divide an amount and clearly show a remainder. Children to represent the lollipop sticks pictorially. There are 3 whole squares, with 1 left over.	13 ÷ 4 = 3 remainder 1 Children should be encouraged to use their times table facts; they could also represent repeated addition on a number line. '3 groups of 4, with 1 left over'				





<u>DIVISION</u>						
Key Language: share group, divide, divided by, half						
Concrete	Pictorial	Abstract				
Sharing using place value counters. 42 ÷ 3 = 14 10s	Children to represent the place value counters pictorially.	Children to be able to make sense of the place value counters and write calculations to show the process. 42 ÷ 3 42 = 30 + 12 30 ÷ 3 = 10 12 ÷ 3 = 4 10 + 4 = 14				
Short division using place value counters to group. 615 ÷ 5 Make 615 with place value counters. How many groups of 5 hundreds can you make with 6 hundred counters? Exchange 1 hundred for 10 tens. How many groups of 5 tens can you make with 11 ten counters? Exchange 1 ten for 10 ones. How many groups of 5 ones can you make with 15 ones?	Represent the place value counters pictorially.	Children to the calculation using the short division scaffold. $\frac{123}{5^{1}15^{5}}$ Begin with divisions that divide equally with no remainder. Move onto divisions with a remainder. Finally move into decimal places to divide the total accurately.				





Key Language: share group, divide, divided by, half							
Concrete	Pictorial	Abstract					
Long division using place value counters 2544 ÷ 12 We can't group 2 thousands into groups of 12 so we exchang We can group 24 hundreds into groups of 12 which leaves us hundred.	1000 100 10 1	$ \begin{array}{r} $					
After exchanging the hundred, we have 14 tens. We can groutens into a group of 12, which leaves 2 tens.	1000s 100s 10s 1s	12 2544 24					
After exchanging the 2 tens, we have 24 ones. We can group into 2 groups of 12, which leaves no remainder.	24 ones 100s 10s 1s	14 12 24 24 0					





<u>Division</u> Key Language: share group, divide, divided by, half						
Concrete		Pictorial		Abstract		
Conceptual variation; different ways to ask children to solve 615 ÷ 5						
Using the part whole model below, how can you divide 615 by 5 without using short division? 615 15	I have £615 and share it equally between 5 bank accounts. How much will be in each account? 615 pupils need to be put into 5 groups. How many will be in each group?		5 615 615 ÷ 5 = = 615 ÷ 5	What is the ca	culation? What	is the answer?





Appendix A – Mathematical Language

High expectations of the mathematical language used are essential, with staff only accepting what is correct. Consistency across the school is key:

Correct Terminology	Incorrect Terminology		
Ones	Units		
Is equal to (is the same as)	Equals		
Zero	Oh (the letter o)		
Exchange	Stealing		
Exchanging	Borrowing		
Regrouping			
Calculation	Generic term of "sum"		
Equation			
Number Sentence			
Known	RUCSAC method to solve problems		
Unknown	(Read, underline, calculate, solve, answer, check)		
Whole			
Part			





Addend - A number to be added to another.

Aggregation - combining two or more quantities or measures to find a total.

Augmentation - increasing a quantity or measure by another quantity.

Commutative – numbers can be added in any order.

Complement – in addition, a number and its complement make a total e.g. 300 is the complement to 700 to make 1,000

Difference – the numerical difference between two numbers is found by comparing the quantity in each group.

Exchange – Change a number or expression for another of an equal value.

Minuend – A quantity or number from which another is subtracted.

Partitioning – Splitting a number into its component parts.

Reduction - Subtraction as take away.

Subitise – Instantly recognise the number of objects in a small group without needing to count.

Subtrahend - A number to be subtracted from another.

Sum - The result of an addition.

Total - The aggregate or the sum found by addition.